Gluon Polarization in the Nucleon

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Outline:

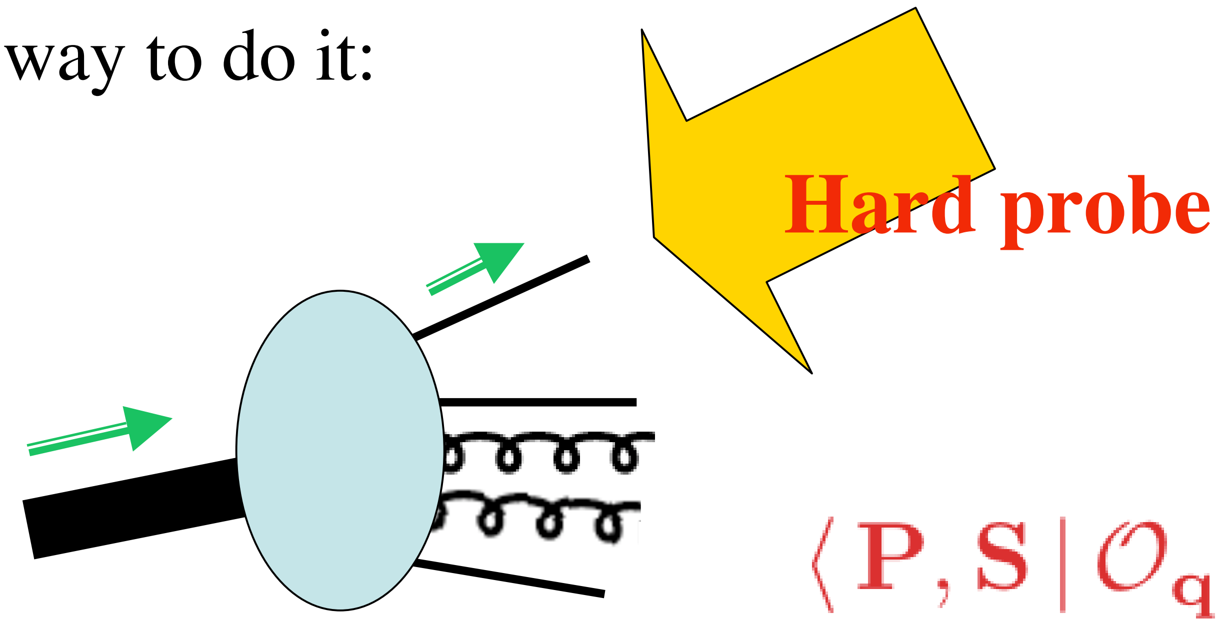
- The parton distribution $\Delta g(x,Q^2)$
- Δg and the proton spin
- Δg and the axial anomaly
- Expectations for Δg
- Phenomenology of Δg in pol. high-energy scatt.
- Conclusions

The parton distribution $\Delta g(x,Q^2)$

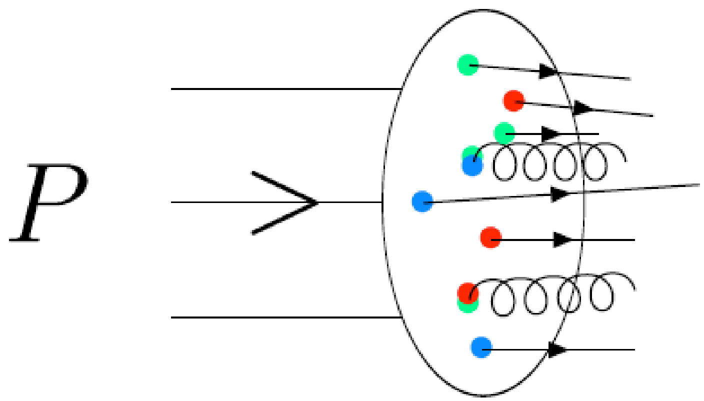
- Main goal of QCD spin physics:

To understand the spin structure of hadrons in terms of quarks and gluons

- The way to do it:



$$\langle \mathbf{P}, \mathbf{S} \,|\, \mathcal{O}_{\mathbf{q},\mathbf{g}} \,|\, \mathbf{P}, \mathbf{S} \rangle$$



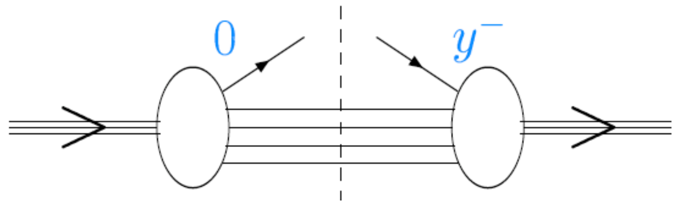
$$P^+ = P_0 + P_3$$

 # density of quarks of given type with momentum fraction between xP^+ **and** $(x+dx)P^+$ **:**

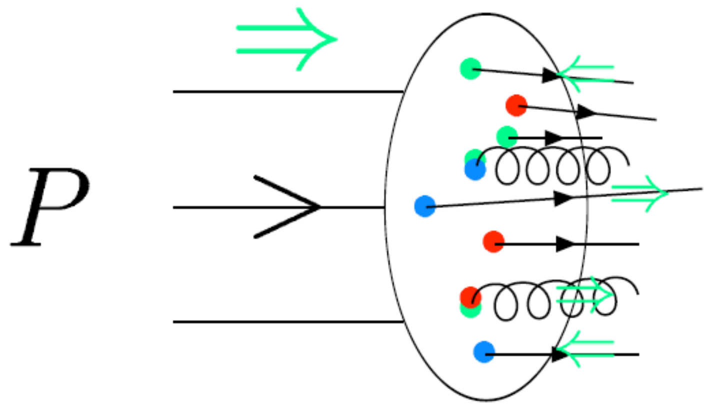
$$q(x) \;\propto\; \int d^2\mathbf{k}_\perp \,\langle\, P \,|\, b^\dagger(xP,\mathbf{k}_\perp)\; b(xP,\mathbf{k}_\perp) \,|\, P \,\rangle$$

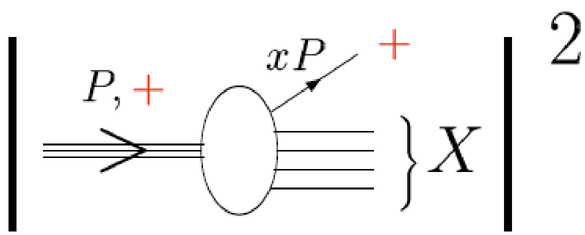
find:

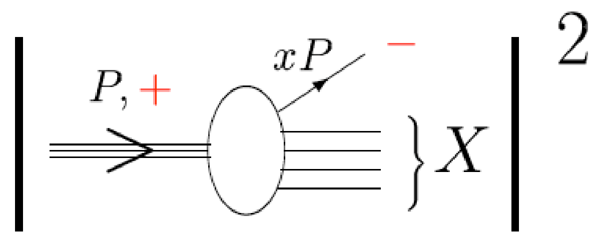
$$q(x) \;=\; \frac{1}{4\pi}\int dy^-\, \mathrm{e}^{-iy^- xP^+}\,\langle\, P \,|\, \bar\psi\,(0,y^-,\mathbf{0}_\perp)\;\gamma^+\,\psi(0)\,|\, P\,\rangle$$

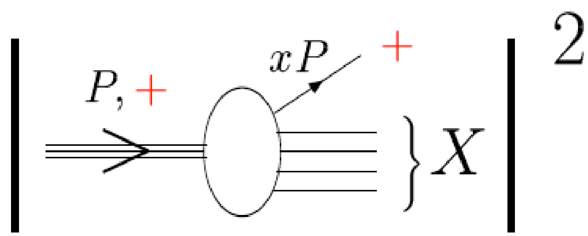


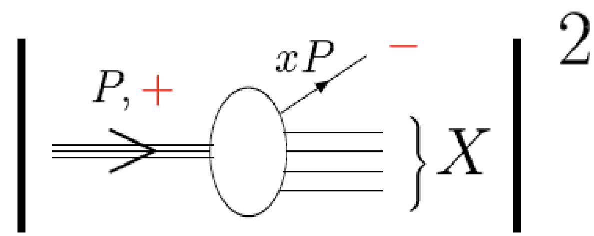
Collins, Soper '81



$q(x) =$ 

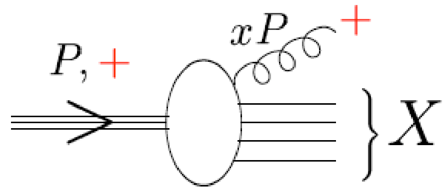
$+$ 

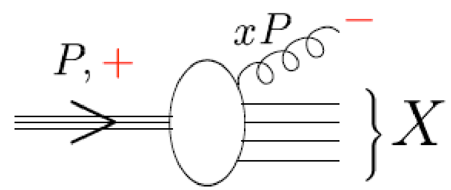
$\Delta q(x) =$ 

$-$ 

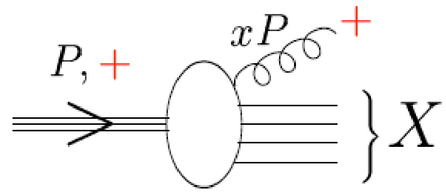
$$\Delta\, q(x) \;=\; \frac{1}{4\pi}\int dy^-\, \mathrm{e}^{-iy^-xP^+}\,\langle\, P,S\,|\,\bar{\psi}\,(0,y^-,\mathbf{0}_\perp)\;\gamma^+\gamma_5\,\psi(0)\,|\,P,S\,\rangle$$

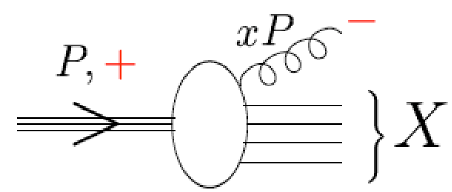
Gluon distributions:

$g(x) = \Big|$ 

$\Big|^2 + \Big|$ 

$\Big|^2$

$\Delta g(x) = \Big|$ 

$\Big|^2 - \Big|$ 

$\Big|^2$

$$\Delta g(x) = \frac{1}{4\pi x P^+} \int dy^- \, \mathrm{e}^{-iy^- xP^+} \langle P, S \,|\, F^{+\alpha}(0, y^-, \mathbf{0}_\perp) \; \tilde{F}_\alpha^{\;+}(0) \,|\, P, S \rangle$$

Collins, Soper; Manohar

This is the quantity that appears in pol. high-energy scattering !

Properties of parton distributions:

- all these operators need renormalization
  → scale dependence / "evolution" of parton distributions
- gauge-invariant when Wilson-line is included :

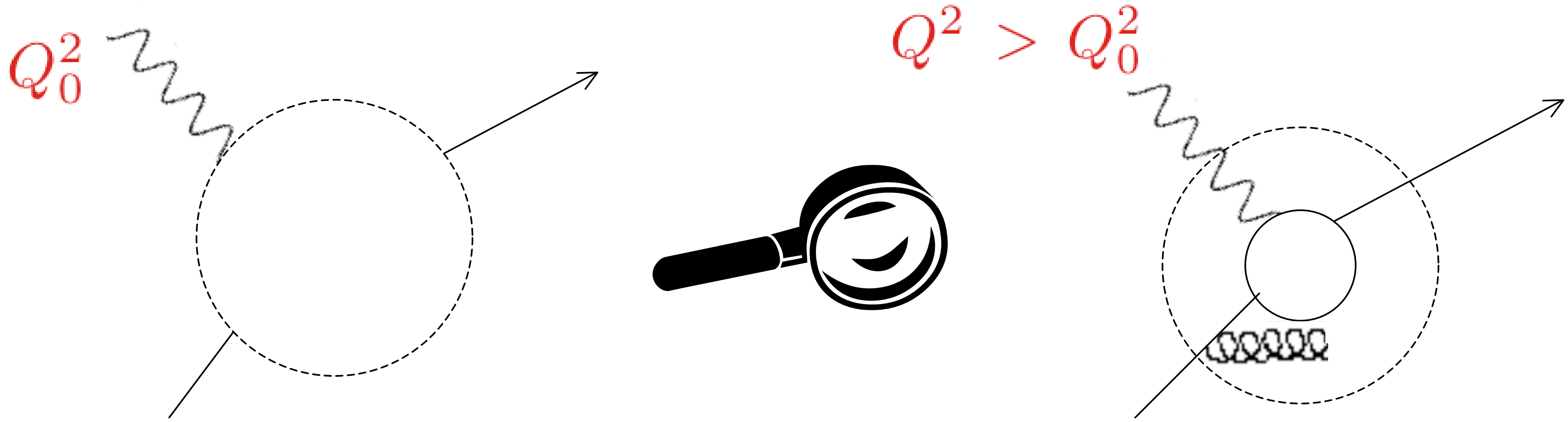
$$\mathcal{P}\exp\left(ig \int_0^{y^-} d\eta\, A^+(\eta) \right)$$

in the following will choose $A^+ = 0$ gauge.

- twist-2
- moments become local operators.
  1st moment of Δg is local only in $A^+ = 0$ gauge:

$$\Delta G(Q^2) = \int_0^1 dx\, \Delta g(x, Q^2) = \frac{1}{2P^+} \langle P, S \,|\, A^1 F^{+2} - A^2 F^{+1} \,|\, P, S \rangle_{Q^2}$$

Jaffe, Manohar '90

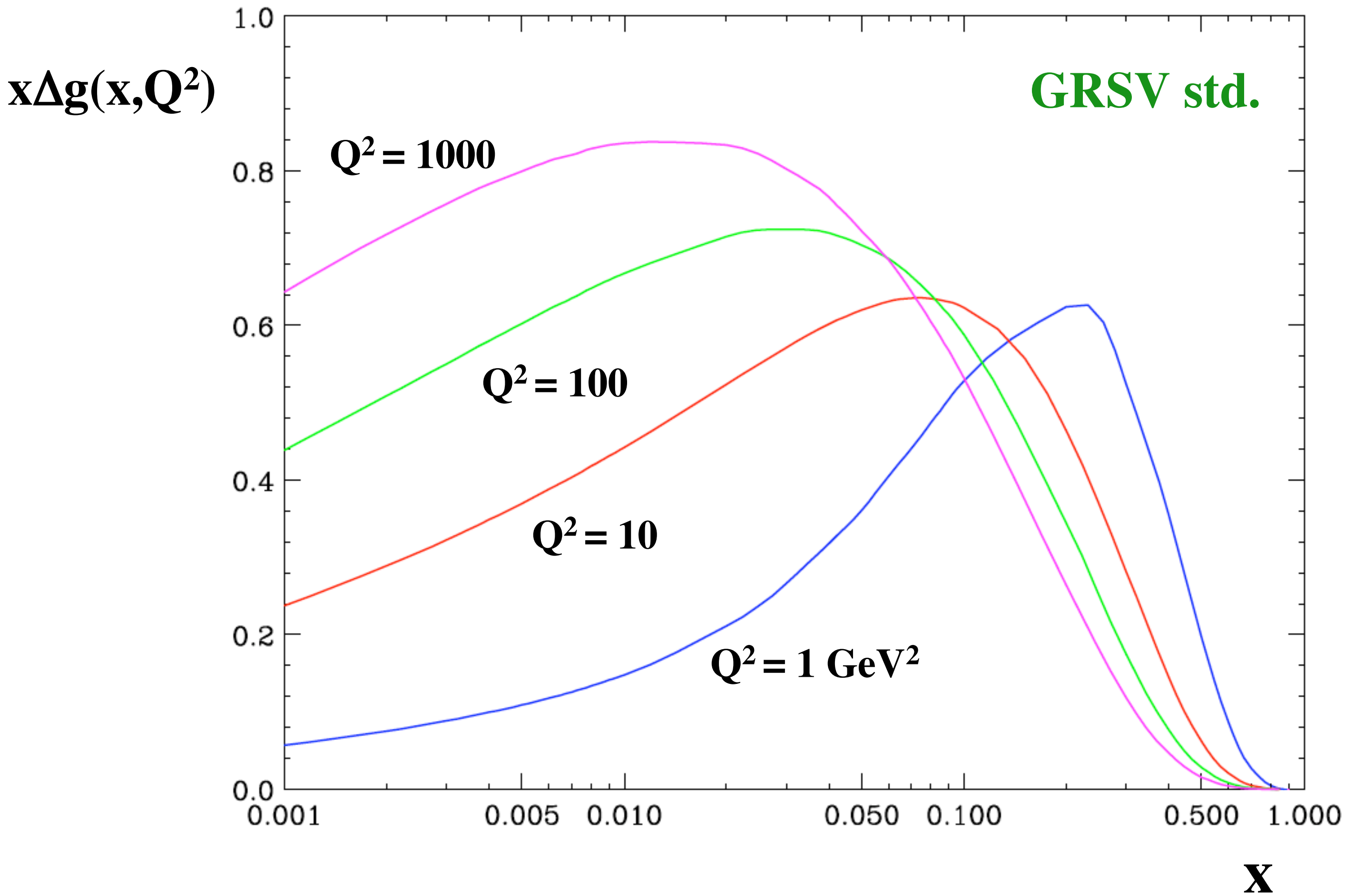


DGLAP evolution equations

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\begin{pmatrix}\Delta q(x,\mu^2)\\ \Delta g(x,\mu^2)\end{pmatrix} = \int_x^1 \frac{dz}{z}\begin{pmatrix}\Delta\mathcal{P}_{qq} & \Delta\mathcal{P}_{qg}\\ \Delta\mathcal{P}_{gq} & \Delta\mathcal{P}_{gg}\end{pmatrix}_{(z,\alpha_s(\mu))}\cdot\begin{pmatrix}\Delta q\\ \Delta g\end{pmatrix}\left(\frac{x}{z},\mu^2\right)$$

$$\Delta\mathcal{P}_{ij}(z,\alpha_s) = \underbrace{\frac{\alpha_s}{2\pi}\Delta P_{ij}^{(0)}(z)}_{\text{"LO"}} + \underbrace{\left(\frac{\alpha_s}{2\pi}\right)^2\Delta P_{ij}^{(1)}(z)}_{\text{"NLO"}} + \dots$$

(Ahmed,Ross; Altarelli,Parisi; Mertig,van Neerven; WV)



Glück, Reya, Stratmann, WV ‘96 / ‘00

Evolution of first moment:

$$\frac{d}{d\log(Q^2)}\begin{pmatrix}\Delta\Sigma\\ \Delta G\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix}0 & 0\\ \frac{3}{2}C_F & \frac{\beta_0}{2}\end{pmatrix}\begin{pmatrix}\Delta\Sigma\\ \Delta G\end{pmatrix}$$

$$\Delta\Sigma = \int_0^1 dx\,\left[\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right](x,Q^2)$$

= twice quark spin contribution
= const.

$$\Delta G(Q^2) = \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\Delta G(Q_0^2) + \frac{3C_F}{\beta_0}\left(\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} - 1\right)\Delta\Sigma$$

Increase ~Log(Q^2) as you look "deeper into the proton" !

$\Delta g(x,Q^2)$ and the proton spin

$$\mathbf{\frac{1}{2} = \langle P, \frac{1}{2} \,|\, J_3 \,|\, P, \frac{1}{2} \rangle}$$

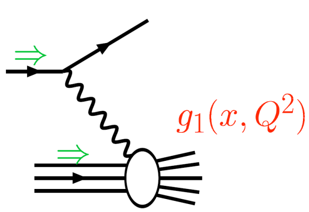
Jaffe, Manohar; Ratcliffe; Ji; Teryaev; Jaffe, Bashinskii; Ji, Hoodbhoy, Lu; …

where $$J_3 = -\frac{1}{2}\,\epsilon_{123} \int d^3x \left[x^1 T^{+2} - x^2 T^{+1} \right]$$

$$= \int d^3x \left[\frac{1}{2}\, q_+^\dagger \gamma_5\, q_+ + \frac{1}{2}\, q_+^\dagger \left(\vec{x} \times \vec{\nabla}\right)_3 q_+ + \left[A^1 F^{+2} - A^2 F^{+1}\right] + F^{+j} \left(\vec{x} \times \vec{\nabla}\right)_3 A^j \right]$$

This gives a sum rule for the proton spin :

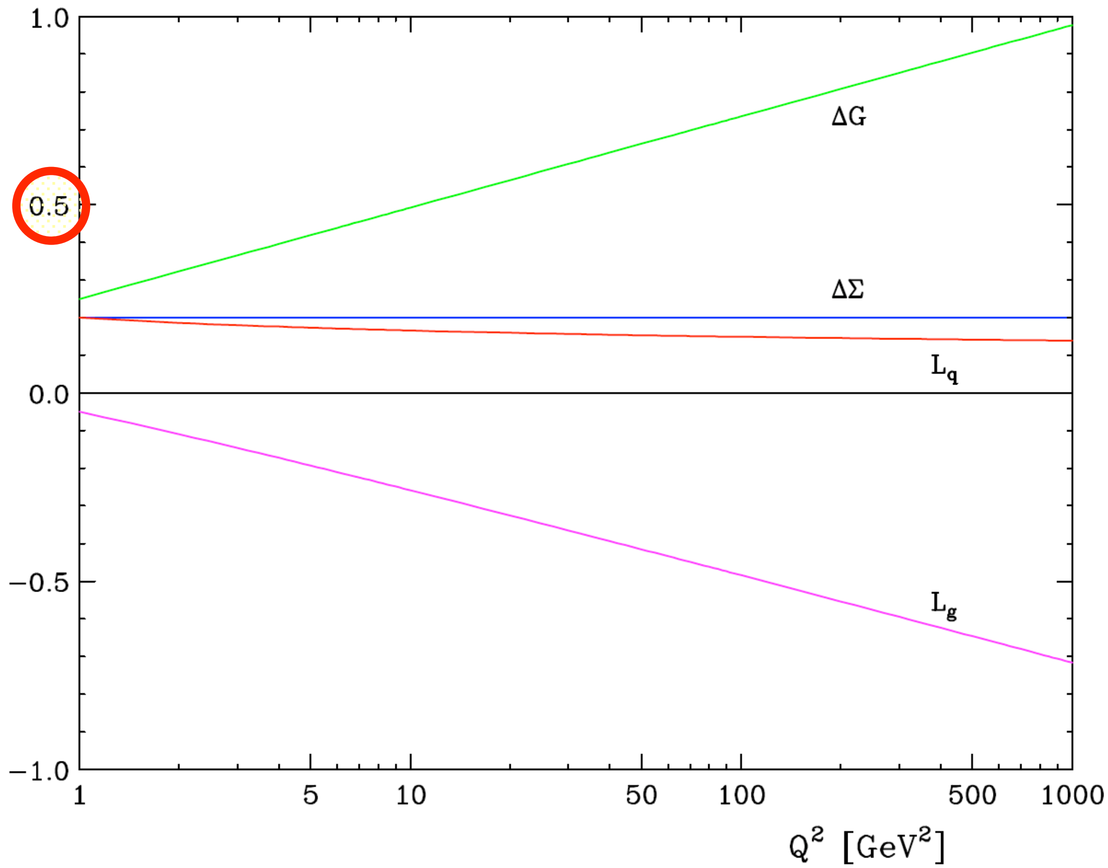
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$



Quark spin
≈ 0.1

EMC, SMC, E142-155, HERMES

LO evolution: **Ratcliffe; Ji, Tang, Hoodbhoy; Hägler, Schäfer**



Asymptotically :

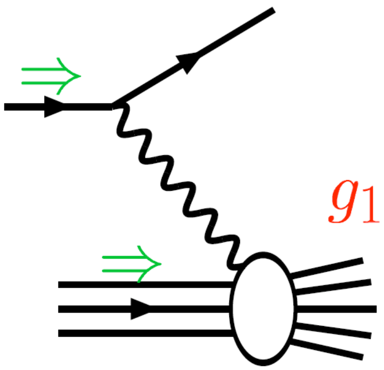
$$J_q = L_q + \frac{1}{2}\Delta\Sigma \to \frac{1}{2}\,\frac{3n_f}{16+3n_f}$$

$$J_g = L_g + \Delta G \to \frac{1}{2}\,\frac{16}{16+3n_f}$$

Ji

Δg and the axial anomaly

How does the QCD axial anomaly enter nucleon spin structure ?



$$\int_0^1 dx\, g_1(x,Q^2) \;\propto\; \frac{1}{9}\,a_0 \;+\; \frac{1}{12}\,a_3 \;+\; \frac{1}{36}\,a_8$$

Proton axial charges

non-singlet : $$a_{3,8} \;\propto\; \langle\, P,S\,|\, \underbrace{\bar{\psi}_q\, \gamma^\mu\, \gamma_5\, \lambda_{3,8}\, \psi_q}_{} \,|\, P,S\,\rangle$$

associated currents *conserved* : $$\partial_\mu\, j^\mu_{3,8} \;=\; 0$$

$$a_{3,8} \;=\; a_{3,8}(\cancel{Q^2})$$

fixed by baryon β-decay constants

Bjorken

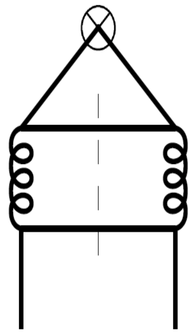
singlet : $$a_0 \propto \langle P, S \,|\, \bar{\psi}_q \, \gamma^\mu \, \gamma_5 \, \mathbb{1} \, \psi_q \,|\, P, S \rangle$$

$$\partial_\mu \, j_0^\mu \;=\; n_f \frac{\alpha_s}{\pi} \, \mathrm{Tr} \left[F_{\mu\nu} \, \tilde{F}^{\mu\nu} \right]$$

due to axial anomaly

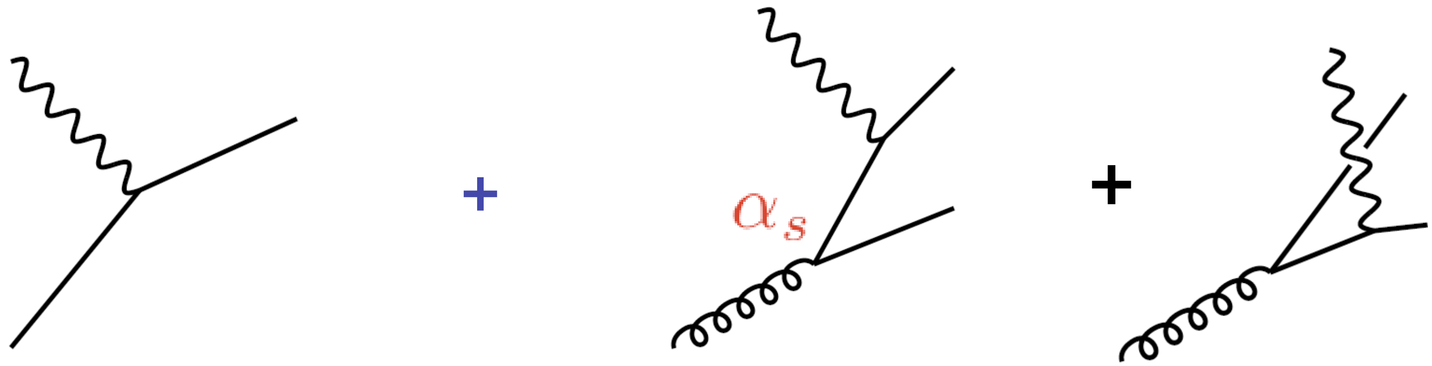
as a result :

$$\frac{d \, a_0(Q^2)}{d \log(Q^2)} \;=\; \alpha_s^2 \; \gamma \; a_0(Q^2)$$



Kodaira; Jaffe; …

Proposal by Altarelli, Ross; Altarelli, Stirling; Efremov, Teryaev; …



$\Delta\Sigma$ ΔG

They found

$$\Delta\Sigma \; - \; \underbrace{n_f \frac{\alpha_s(Q^2)}{2\pi} \Delta G(Q^2)}_{\text{does not vanish at large } Q^2 \text{ !}}$$

- **indeed** $$\frac{d\left[\Delta\Sigma - n_f \frac{\alpha_s}{2\pi}\Delta G\right]}{d\log(Q^2)} = \alpha_s^2 \, \gamma \left[\Delta\Sigma - n_f \frac{\alpha_s}{2\pi}\Delta G\right]$$

- **idea is that axial charge can be small if ΔG is large**

How large ?

- **early estimates (late '80s) :**

$$0 \;=\; 0.6 \;-\; 3\,\frac{\alpha_s(Q^2)}{2\pi}\,\Delta G(Q^2)$$

$$\longrightarrow\;\; \Delta G(Q^2 = 4\,\mathrm{GeV}^2) \;\approx\; 4 \;-\; 5$$

- **"more modern" :**

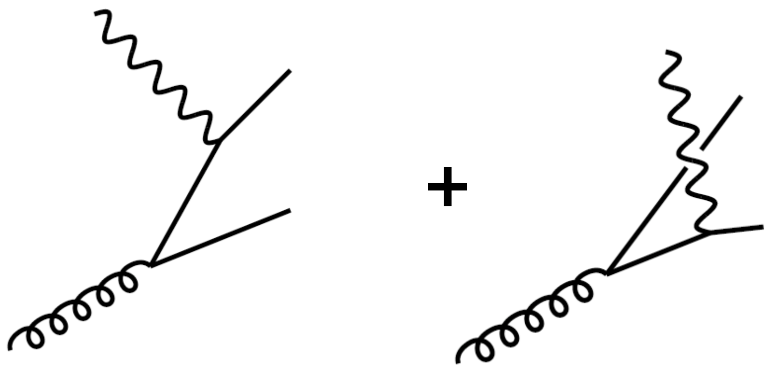
$$0.25\left(1 \;-\; \frac{\alpha_s(Q^2)}{\pi}\right) \;=\; 0.6\left(1 \;-\; \frac{\alpha_s(Q^2)}{\pi}\right) \;-\; 3\,\frac{\alpha_s(Q^2)}{2\pi}\,\Delta G(Q^2)$$

$$\longrightarrow\;\; \Delta G(Q^2 = 4\,\mathrm{GeV}^2) \;\approx\; 1.5 \;-\; 2$$

- “perturbative-anomaly” scenario controversial in the literature

Jaffe, Manohar

- result for “photon-gluon fusion” process depends on scheme



result $-n_f \frac{\alpha_s}{2\pi} \Delta G$ emerges from large-k_T part of diagrams

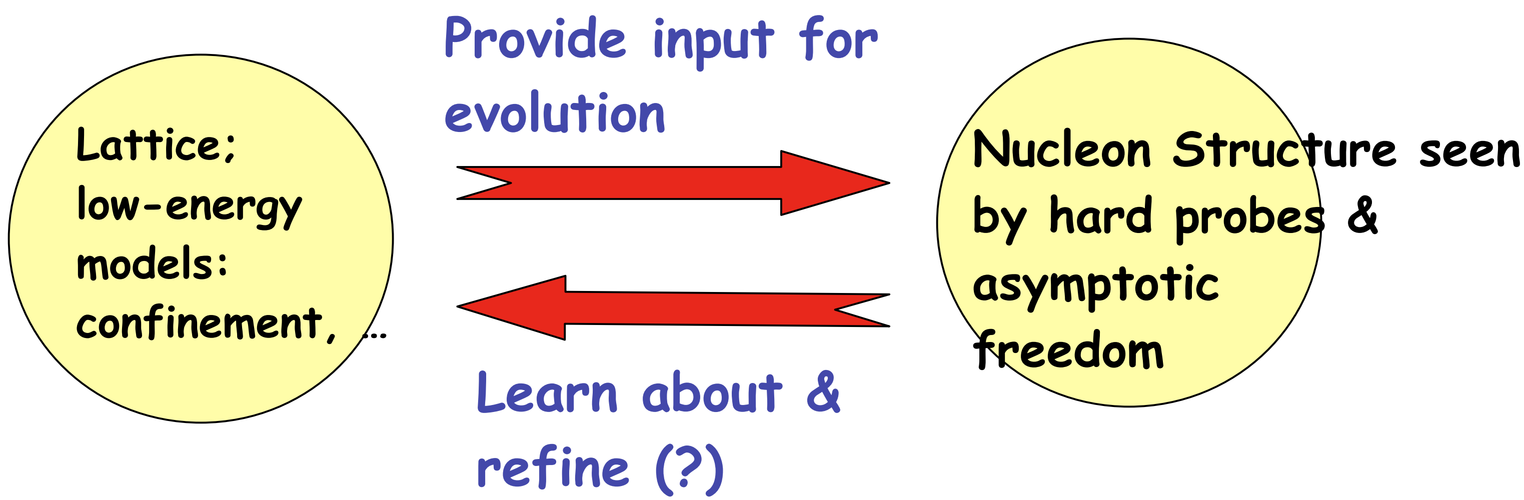
Carlitz, Collins, Mueller; Bass, Thomas; Mankiewicz, Schäfer; Ellwanger; WV

- is huge gluon spin contribution better than small quark one …?

- in any case, has given boost to attempts to measure ΔG

Expectations for Δg

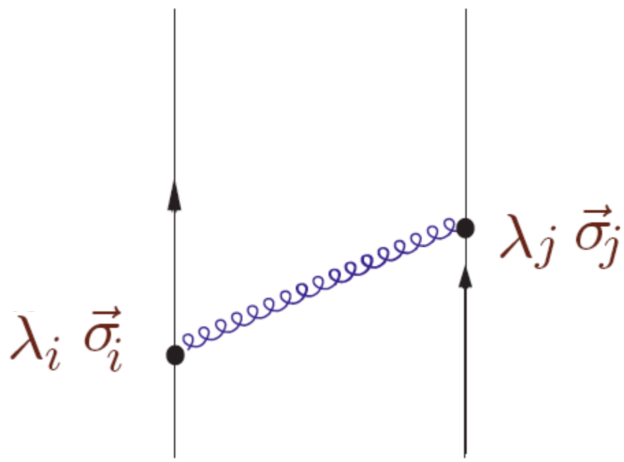
Ideally …



- there have been a number of attempts to estimate ΔG in models and on the lattice, mostly to see wether a very large ΔG is possible or even likely
- so far, have rather limited insights
- hope is that much of this will be revisited, now that measurements are becoming available !

- **constituent quark model estimates :** Jaffe; Barone, Calarco, Drago

$$\Delta G = \langle p \uparrow | \int d^3x \, 2\mathrm{Tr}\left\{ \left(\vec{E} \times \vec{A} \right)_z + \vec{A}_\perp \cdot \vec{B}_\perp \right\} | p \uparrow \rangle$$



→ ambient gluon fields

Typically, Hamiltonian contains term

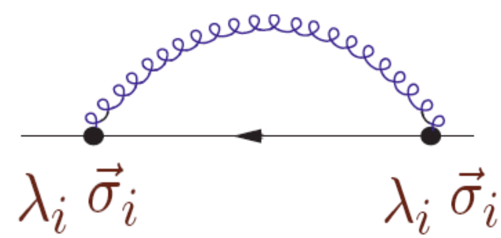
$$\alpha_s \sum_{i \neq j} \lambda_i^a \, \lambda_j^a \, \vec{\sigma}_i \cdot \vec{\sigma}_j$$

$\alpha_s = 0.9$ **determined from N-Δ mass splitting**

sets scale for model : $Q_0 \approx 0.5$ GeV

- **Jaffe :** $\Delta G(Q_0^2) \;=\; -0.8$
- **Barone, Calarco, Drago :** $\Delta G(Q_0^2) \;=\; 0.24$

Isgur-Karl model. Sign change because of "self interactions"



- **Bag model estimates :** **Jaffe; Lee, Min, Park, Rho, Vento**

  Confining boundary condition for color electric field plays important role

  - **Jaffe :** $\Delta G(Q_0^2) \;=\; -0.2$
  - **Lee et al. :** $\Delta G(Q_0^2) \;\approx\; 0.2$

- **QCD sum rules :** $\Delta G(1\ \mathrm{GeV}^2) \;\approx\; 2 \pm 1$ **Mankiewicz, Piller, Saalfeld**

- **Helicity retention, color coherence :** $\Delta G(1\ \mathrm{GeV}^2) \;\approx\; 0.5$

  Brodsky, Burkardt, Schmidt

- **ΔG on the lattice ?**

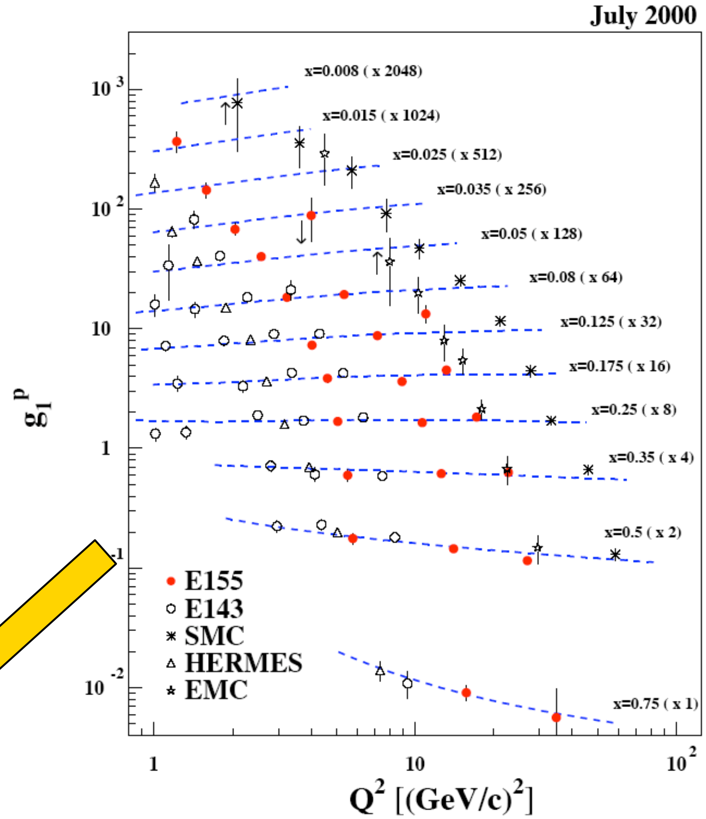
  Non-local operator, except for $A^+ = 0$ gauge …

Attempt by Mandula '90 :

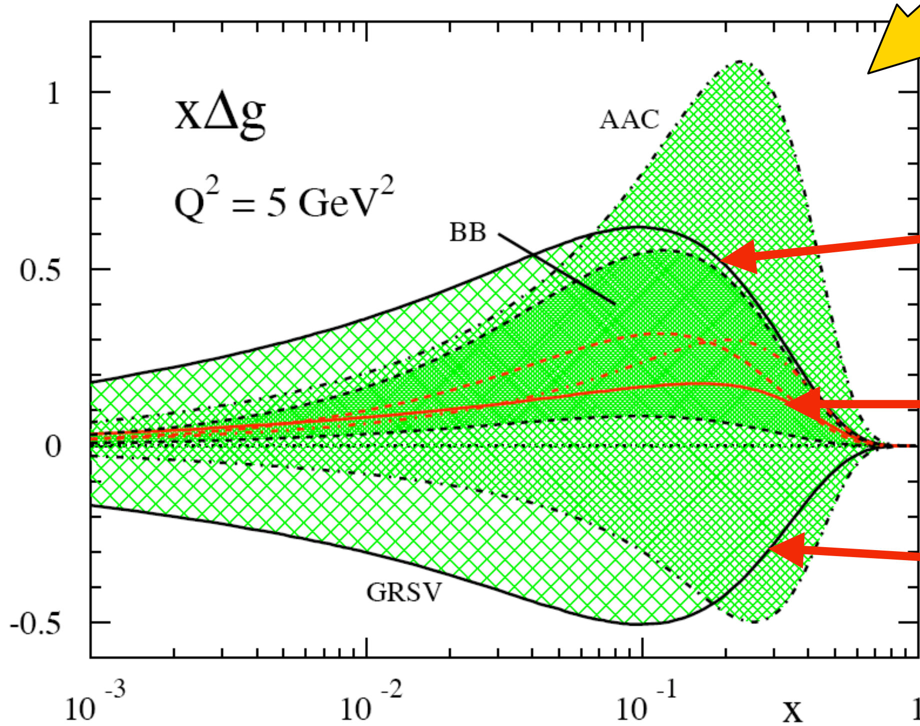
$$3\frac{\alpha_s}{2\pi}\Delta G \;\leq\; 0.05$$

Phenomenology of Δg in polarized high-energy scattering

- **Δg contributes to scaling violations of $g_1(x,Q^2)$**



also: Gehrmann, Stirling; Altarelli, Ball, Forte, Ridolfi; SMC; E155; de Florian, Navarro, Sassot; Leader, Sidorov, Stamenov; …



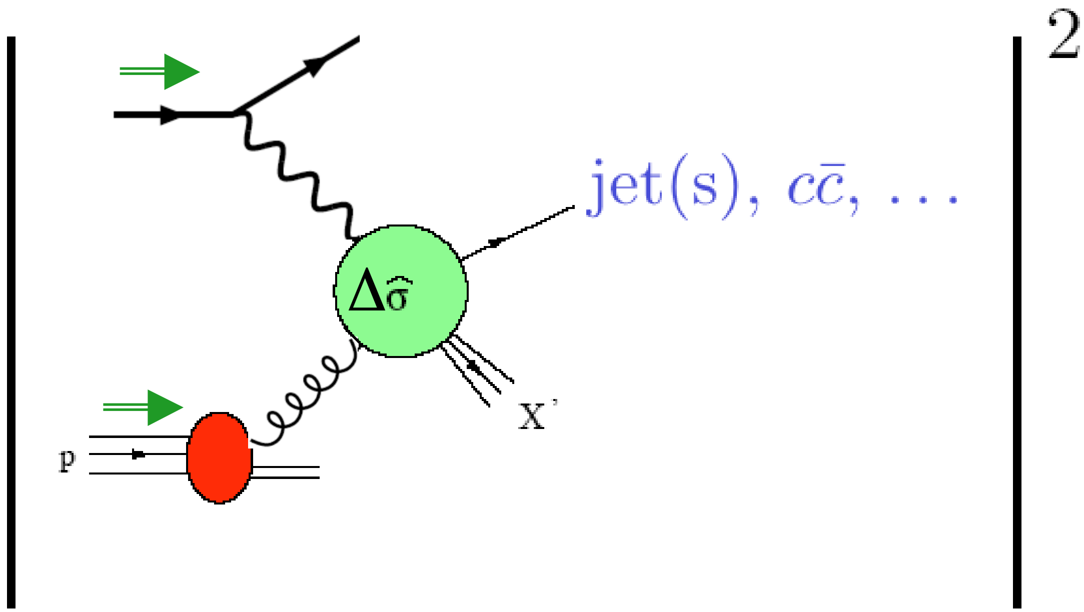
$$\Delta G(1\,\mathrm{GeV}^2) \approx 1.8$$

$$\Delta G(1\,\mathrm{GeV}^2) \approx 0.4$$

$$\Delta G(1\,\mathrm{GeV}^2) \approx -1.7$$

- **Δg can be probed in other reactions in polarized high-energy scattering :**

lepton scattering



$$p_T^3 \frac{d\Delta\sigma^{pp\to \mathrm{F}X}}{dp_T d\eta} = \sum_{abc} \int dx_a \, dx_b \, \Delta f_a(x_a, \mu) \, \Delta f_b(x_b, \mu) \times p_T^3 \frac{d\Delta\hat{\sigma}^{ab\to \mathrm{F}X'}}{dp_T d\eta}(x_a P_a, x_b P_b, P^{\mathrm{F}}, \mu) \; + \; \text{P.C.}$$

$$\Delta\hat{\sigma}^{(0)} + \alpha_s \Delta\hat{\sigma}^{(1)} + \ldots$$ perturb.

- important to have adequate (and proven) theoretical framework :

    - higher-order (NLO) corrections

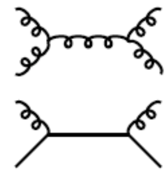
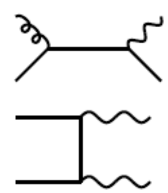
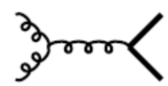
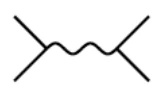
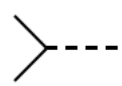
        - * often sizable, reduce scale dependence

    - comparison to data for unpolarized cross sections

- need variety of probes

- eventually, will need a "CTEQ-style" global analysis

RHIC offers best possibilities to probe Δg :

Reaction	Dom. partonic process	probes	LO Feynman diagram
$\vec{p}\vec{p} \to \pi + X$ [61, 62]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	
$\vec{p}\vec{p} \to \mathrm{jet(s)} + X$ [71, 72]	$\vec{g}\vec{g} \to gg$ $\vec{q}\vec{g} \to qg$	Δg	(as above)
$\vec{p}\vec{p} \to \gamma + X$ $\vec{p}\vec{p} \to \gamma + \mathrm{jet} + X$ $\vec{p}\vec{p} \to \gamma\gamma + X$ [67, 73, 74, 75, 76]	$\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{g} \to \gamma q$ $\vec{q}\vec{\bar{q}} \to \gamma\gamma$	Δg Δg $\Delta q, \Delta\bar{q}$	
$\vec{p}\vec{p} \to DX, BX$ [77]	$\vec{g}\vec{g} \to c\bar{c}, b\bar{b}$	Δg	
$\vec{p}\vec{p} \to \mu^+\mu^- X$ (Drell-Yan) [78, 79, 80]	$\vec{q}\vec{\bar{q}} \to \gamma^* \to \mu^+\mu^-$	$\Delta q, \Delta\bar{q}$	
$\vec{p}\vec{p} \to (Z^0, W^\pm)X$ $p\vec{p} \to (Z^0, W^\pm)X$ [78]	$\vec{q}\vec{\bar{q}} \to Z^0$, $\vec{q}'\vec{\bar{q}} \to W^\pm$ $\vec{q}'\bar{q} \to W^\pm$, $q'\vec{\bar{q}} \to W^\pm$	$\Delta q, \Delta\bar{q}$	

Jäger,Schäfer, Stratmann,WV

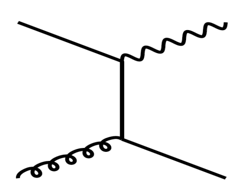
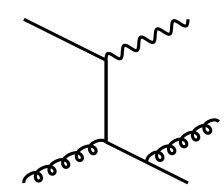
Jäger,Stratmann,WV; Signer et al.

Gordon,WV; Contogouris et al.; Gordon, Coriano

Stratmann, Bojak

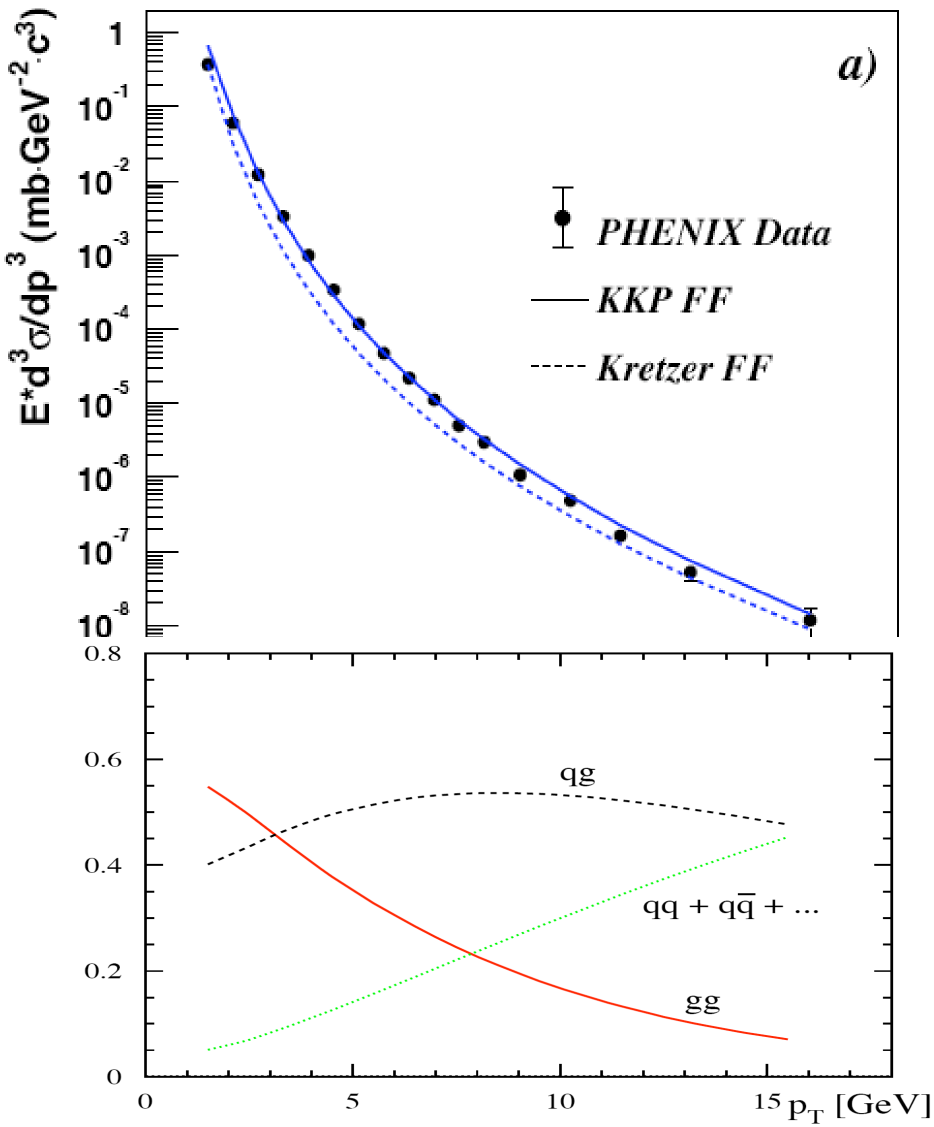
Weber; Gehrmann; Kamal

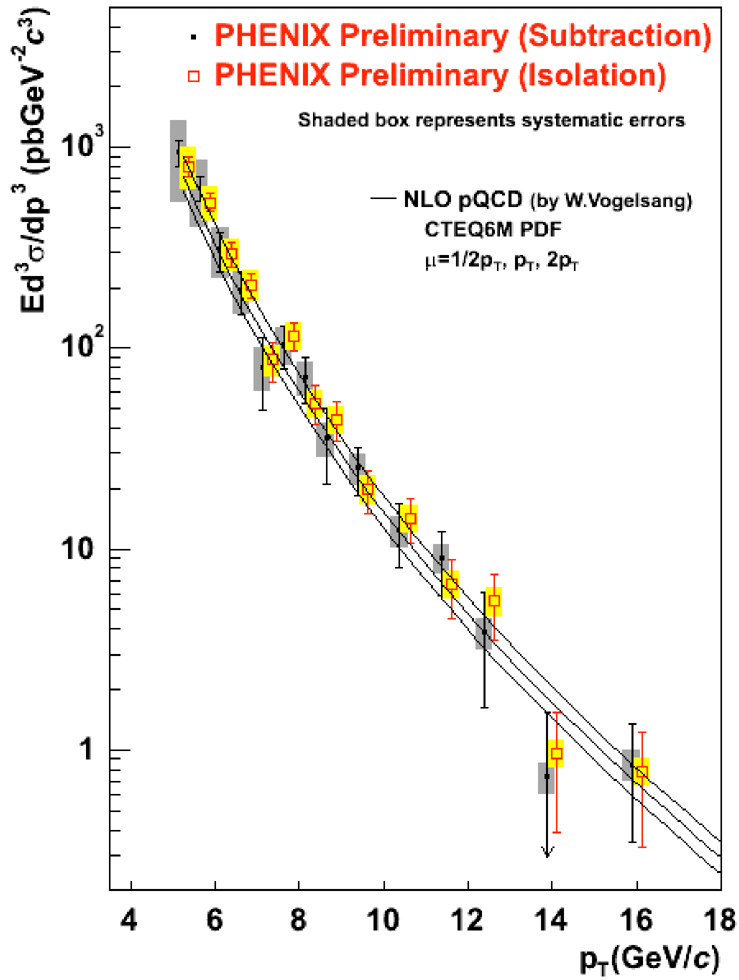
NLO corrections known in all cases.

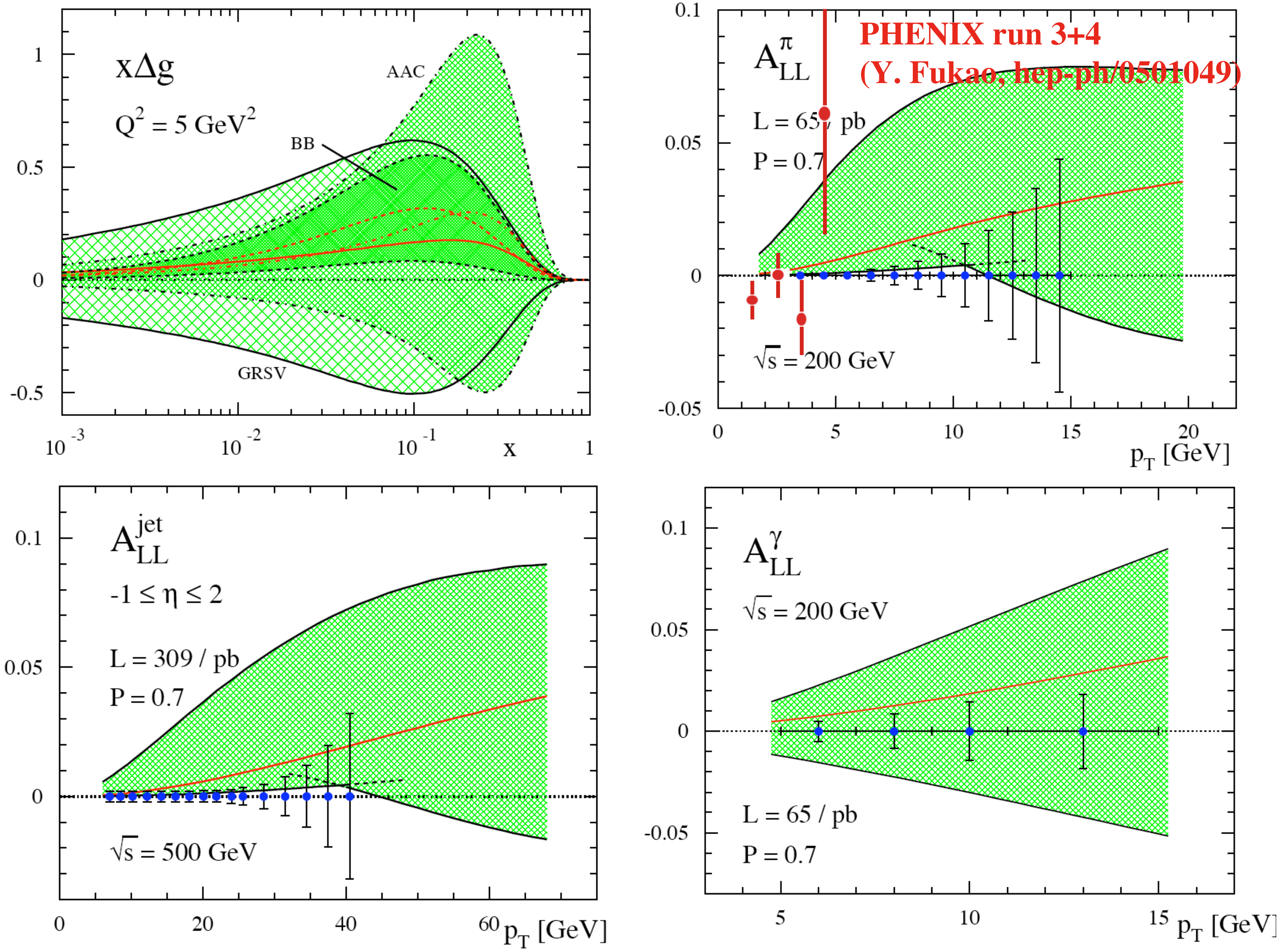



- **Calculations are already successful in unpolarized pp at RHIC:**

$$pp \rightarrow \pi^0 X \qquad\qquad pp \rightarrow \gamma X$$

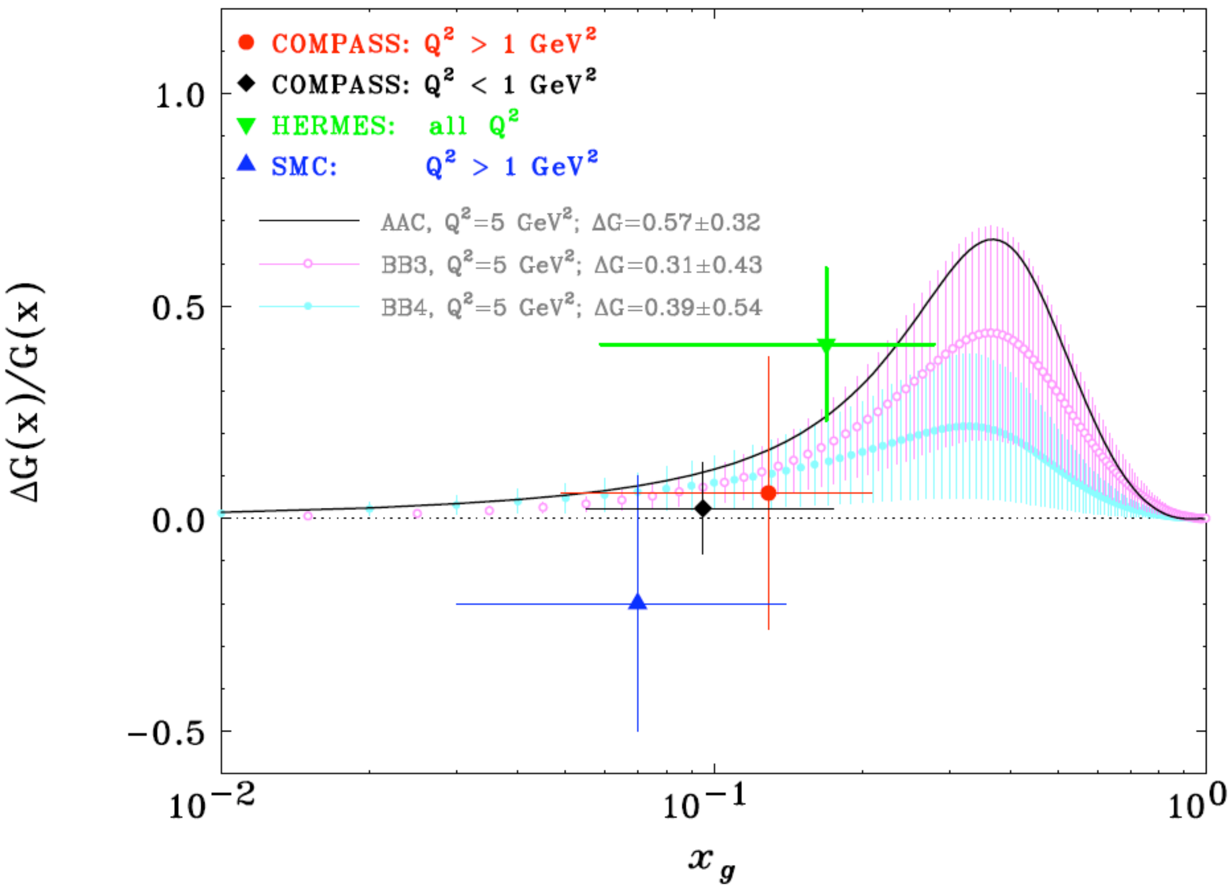






xΔg
Q² = 5 GeV²
AAC
BB
GRSV
x
A_LL^π
PHENIX run 3+4
(Y. Fukao, hep-ph/0501049)
L = 65 / pb
P = 0.7
√s = 200 GeV
p_T [GeV]
A_LL^jet
-1 ≤ η ≤ 2
L = 309 / pb
P = 0.7
√s = 500 GeV
p_T [GeV]
A_LL^γ
√s = 200 GeV
L = 65 / pb
P = 0.7
p_T [GeV]

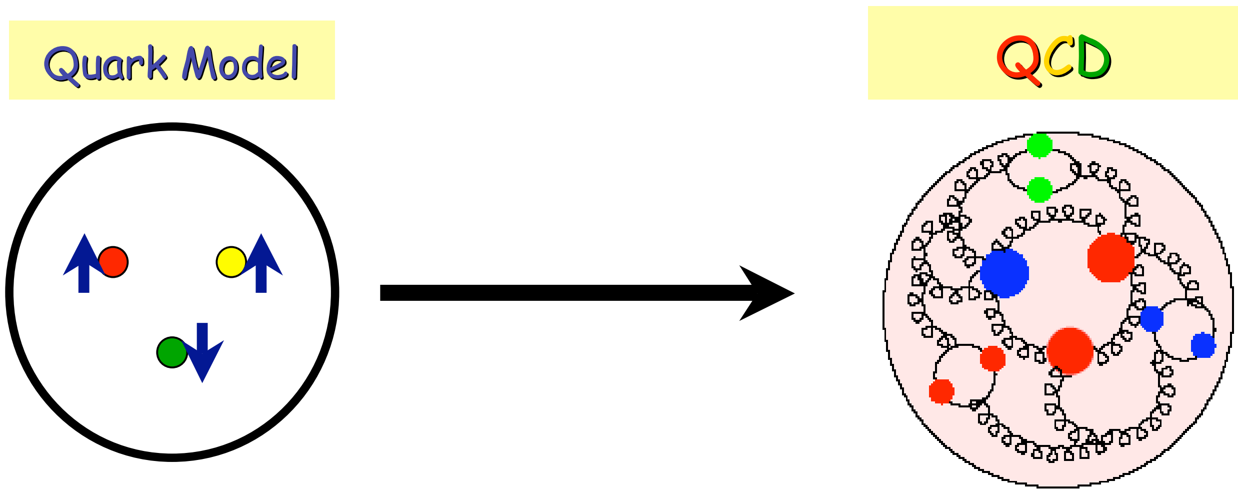
Ellis, Karliner



- Theory good enough ?
- it's a stretch to conclude anything about the integral ΔG …

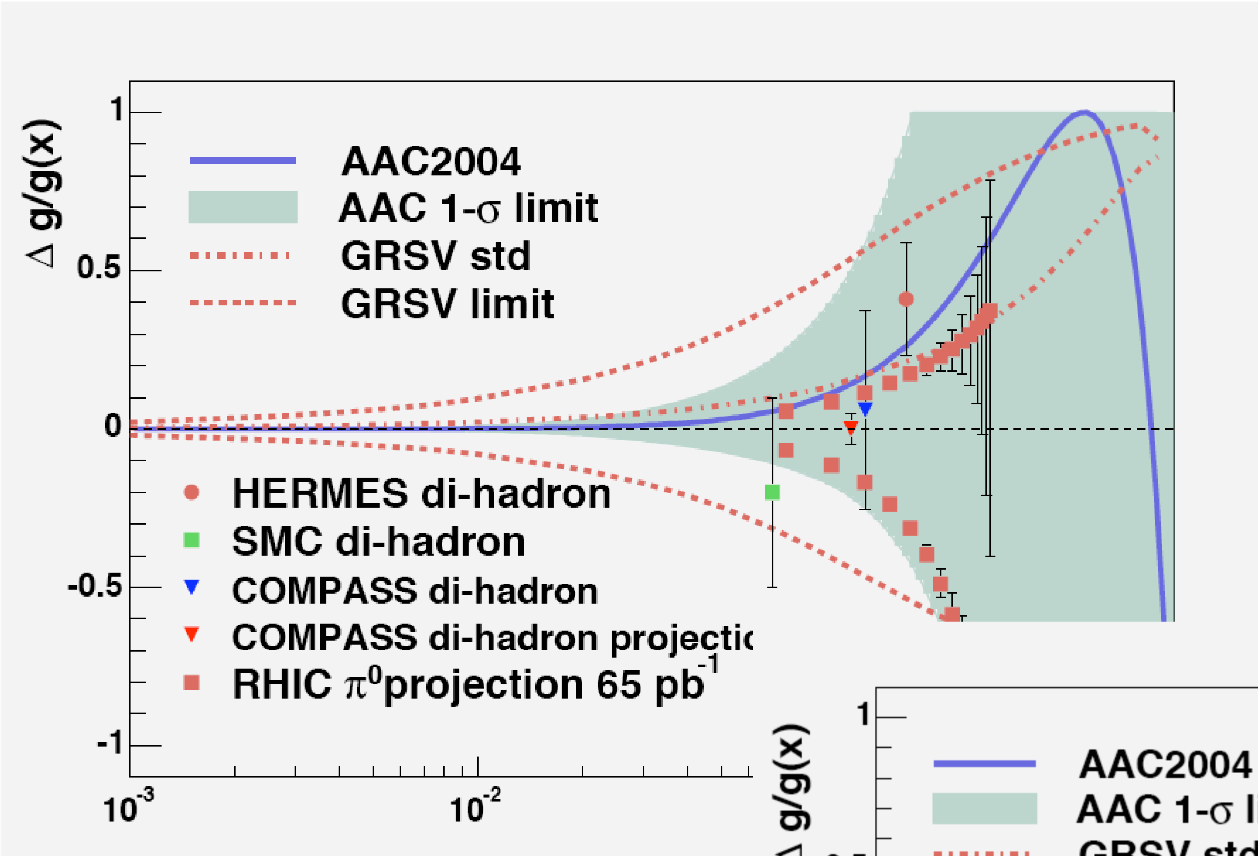
Conclusions & Outlook:

- nucleon spin structure has turned out to be quite subtle …

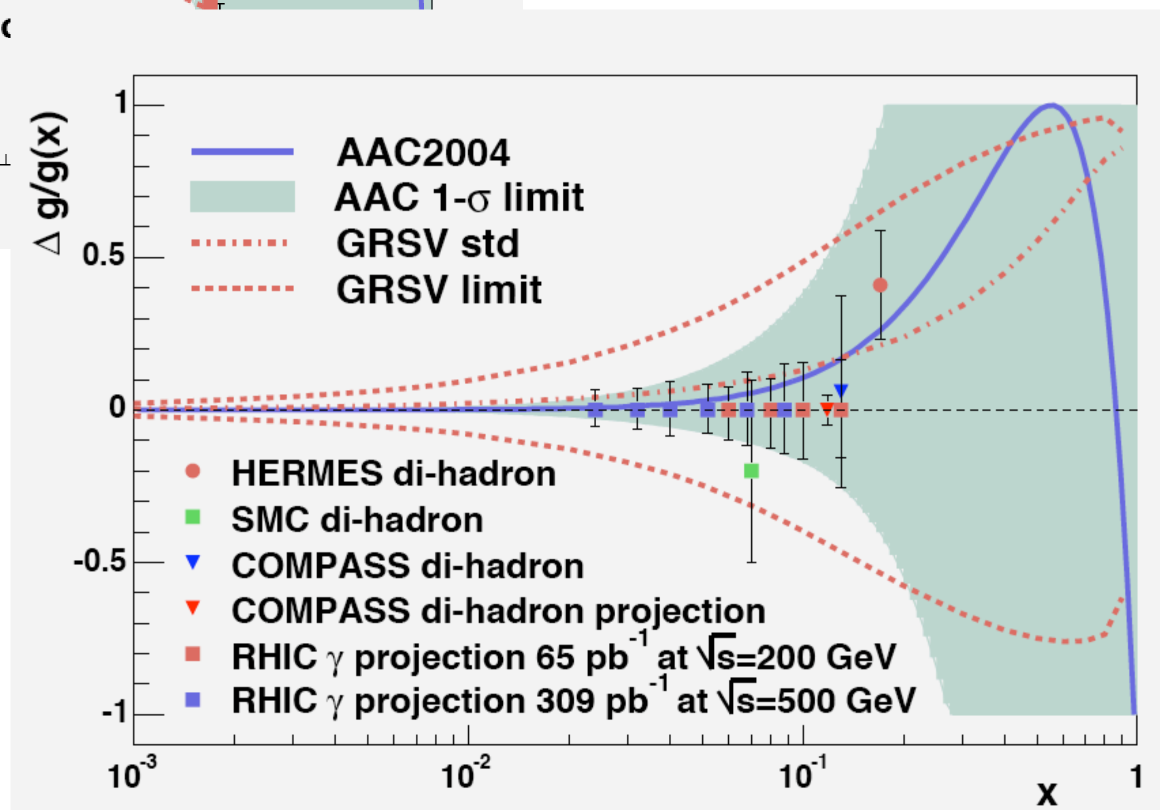


- gluon polarization promises to be an important piece for our understanding
- carried by tremendous efforts in experiments :
  RHIC, HERMES, COMPASS -- eRHIC ?
- complementarity of lepton-nucleon scattering and pp !
- improvements in models, lattice ?

N. Saito



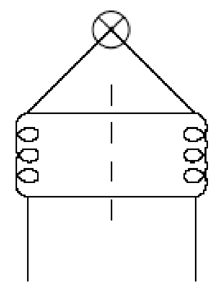
Δ g/g(x)
1
0.5
0
-0.5
-1
10^{-3}
10^{-2}
AAC2004
AAC 1-σ limit
GRSV std
GRSV limit
HERMES di-hadron
SMC di-hadron
COMPASS di-hadron
COMPASS di-hadron projectic
RHIC π^0projection 65 pb^{-1}



Δ g/g(x)
1
0.5
0
-0.5
-1
10^{-3}
10^{-2}
10^{-1}
1
x
AAC2004
AAC 1-σ limit
GRSV std
GRSV limit
HERMES di-hadron
SMC di-hadron
COMPASS di-hadron
COMPASS di-hadron projection
RHIC γ projection 65 pb^{-1} at $\sqrt{s}$=200 GeV
RHIC γ projection 309 pb^{-1} at $\sqrt{s}$=500 GeV

- a crucial feature of the quark singlet :

$$s^\mu \, \Delta\Sigma \;=\; \langle P,S \mid \underbrace{\bar\psi\,\gamma^\mu\,\gamma^5\,\frac{1\!\!1}{2}\,\psi}_{\text{singlet axial current } j_5^{\mu,0}} \mid P,S\rangle$$



- connection to axial anomaly :

$$\partial_\mu \, j_5^{\mu,0} \equiv \partial_\mu \left[\bar\psi\,\gamma^\mu\,\gamma^5\,\psi\right] = n_f\,\frac{\alpha_s}{2\pi}\,\mathrm{Tr}\left[G_{\mu\nu}\,\tilde G^{\mu\nu}\right]$$

- a consequence (in $\overline{\mathrm{MS}}$ scheme) :

$$\Delta\Sigma(Q^2) \;=\; \left(1 \;+\; \frac{6n_f}{(33-2n_f)\,\pi}\;\left[\,\alpha_s(Q^2)\;-\;\alpha_s(\mu_0^2)\,\right]\right)\,\Delta\Sigma(\mu_0^2)$$

$\Rightarrow$ moderate decrease in perturbative region (Jaffe)

$$\partial_\mu\left[j_5^\mu - fK^\mu\right] = 0 \;\;, \qquad K^\mu \equiv \frac{\alpha_s}{4\pi}\,\epsilon^{\mu\nu\rho\sigma} A_\nu^a \left(G_{\rho\sigma}^a - \frac{g}{3} f_{abc} A_\rho^b A_\sigma^c\right)$$

$$j_5^\mu \;=\; \left[\,j_5^\mu - fK^\mu\,\right] \;+\; fK^\mu$$

"AB scheme"